Slide 3: Recap on Logical Implication (Entailment) |-|=-

- Entailment notation: p |=q if and only if the implication p $\implies$ q is a tautology.
- Example:
  - p $\wedge$ q |=q
  - Truth table for p $\implies$ q:

p	q	p $\wedge$ q	p $\implies$ q
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Slide 4: (r $\implies$ s) $\wedge$ (r $\implies$ $\neg$s) |-|=-

- Intuitively, if r implies both s and $\neg$s, then r must be false.
- Truth table for (r $\implies$ s) $\wedge$ (r $\implies$ $\neg$s):

r	s	$\neg$s	(r $\implies$ s) $\wedge$ (r $\implies$ $\neg$s)
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	T

Slide 5: p $\vdash$ q

- Notation: p $\vdash$ q means q is provable from p using inference rules.
- Example:
  - A $\implies$ B, $\neg$A, therefore $\neg$B

Slide 6: Differences Between |-|=- and $\vdash$

- |= indicates semantic entailment (truth conditions).
- $\vdash$ represents syntactic derivation (inference rules).

Slide 7: Recap on Inference Rules

- Example inference rules:
  - Modus Ponens ($\implies$ Elim):
    p $\implies$ q, p $\vdash$ q
  - Conjunction Introduction ($\wedge$Intro):
    p $\vdash$ q, p $\vdash$ r $\vdash$ p $\wedge$ q
  - Conditional Proof ($\implies$ Intro):
    p $\vdash$ r, p $\vdash$ s $\vdash$ p $\implies$ (r $\wedge$ s)

Slide 8: Layout of an Inference Rule

- Premises above the line, conclusion below the line.
- Example inference rule ($\implies$ Intro):

  p ⊢ r, p ⊢ s p $\implies$ (r ∧ s)

Slide 9: Presentation of Proofs

- Steps:
  - Number each step.
  - Justify each step with previous line(s) and inference rule used.

Slide 10: Deriving ¬p $\implies$ r From (p ∧ q) ∨ r

- Example proof:

  (p ∧ q) ∨ r, ¬E ... ¬p $\implies$ r

Slide 11: Two Special Inference Rules

- Deductive Theorem ($\implies$ Intro):

  p ⊢ r, p ⊢ s p $\implies$ (r ∧ s)

- Reductio ad absurdum (¬Intro):

  p ⊢ r, p ⊢ ¬s p ⊢ ¬r

Slide 12: Conditional Proofs

- Strategy: Assume p, deduce q if possible, discharge assumption.
- Example:

  (p ∧ q) ∨ r ... ¬p $\implies$ r

Slide 13: Indirect Proofs

- Strategy: Assume negation of goal, deduce contradiction.
- Example:

  (p ∧ q) ∨ r ... ¬p $\implies$ r

Slide 14: Solution to Exercise

Given argument: A (You eat carefully) B (You have a healthy digestive system) C (You exercise regularly) D (You are very fit) B D E (You live to a ripe old age) ¬E Therefore, ¬A ¬C

Proof:

Line	Formula	Justification
1	A B	Premise
2	C D	Premise
3	B D E	Premise
4	¬E	Premise

Line	Formula	Justification
5	$\neg(B \quad D)$	Modus Tollens (3, 4)
6	$\neg B \quad \neg D$	De Morgan's Law (5)
7	$\neg B$	Elim (6)
8	$\neg A$	Modus Tollens (1, 7)
9	$\neg D$	Elim (6)
10	$\neg C$	Modus Tollens (2, 9)
11	$\neg A \quad \neg C$	Intro (8, 10)

Conclusion: We have proven that $\neg A \quad \neg C$, i.e., you did not eat carefully and you did not exercise regularly.

Slide 15: Two Special Inference Rules (continued)

- Deductive Theorem:

  $p \vdash r, p \vdash s \; p \implies (r \land s)$

- Reductio ad absurdum:

  $p \vdash r, p \vdash \neg s \; p \vdash \neg r$

Slide 16: Soundness and Completeness

- Sound: Valid argument with true premises.
- Complete: Derives any sentence entailed by premises.

Slide 17: Formal Proofs of Natural Language Arguments

- Steps:
  - Identify atomic propositions.
  - Formalize argument in logic.
  - Check for invalidity.
  - Attempt proof.

Slide 18: Example - Travel

- Argument:

  ... Therefore, if my neighbours claim to be impressed then they are just pretending.

Slide 19: Example - Travel (continued)

- Formalize argument:

  $p \implies q, \neg p \implies \neg r, \neg q \dots \neg r$

- Proof:

  $\dots \neg p \implies r$

Slide 20: Example - Nutrition

- Argument:

  … Therefore, you did not eat carefully and you did not exercise regularly.

Slide 21: Example - Nutrition (continued)

- Formalize argument: $A \implies B, C \implies D, B \lor D \implies E, \neg E$ … $\neg A \land \neg C$
- Proof: … $\neg A \land \neg C$

Slide 22: Application to Software Engineering

- Questions about software specifications and claims are arguments.

Slide 23: Reading and References

- Russell and Norvig, Artificial Intelligence (4th Edition)
- Nissanke, Introductory Logic and Sets for Computer Scientists
- Gray, Logic, Algebra and Databases