Week 22 Validity and Inference Rules

Detailed Notes on Lectures 9 & 10: Validity and Inference Rules

Slide 1: Learning Objectives

- Define the notion of validity in an argument.
- Establish validity using truth tables.
- Demonstrate invalidity using truth tables.
- Understand inference rules.

Slide 2: Contents

- Objectives
- Transformational proofs are not sufficient.
- Comparison of deduction with induction.
- Validity.
- Demonstrating validity/invalidity using truth tables.
- Problem with truth tables.
- Inference rules.
- Summary, reading, and references.

Slide 3: Transformational Proofs do not Suffice

- Understanding transformations of formulas is useful but insufficient.
- Logic uses rules of inference to deduce true propositions from other true propositions.
- Invalid premises cannot lead to valid conclusions, preventing proofs of contradictions or useless systems.

Slide 4: Premises and Conclusions

- An argument consists of premises (basis for accepting) and a conclusion.
- Example:
  - Premises: Every adult is eligible to vote; John is an adult.
  - Conclusion: Therefore, John is eligible to vote.

Slide 5: Deduction vs. Induction

- Deductive arguments: Conclusion is wholly justified by premises.
- Inductive arguments: More general new knowledge inferred from facts or observations.

Slide 6: Valid vs. Invalid Arguments

- Valid arguments: Conclusion always true when premises are true.
- Invalid arguments: At least one assignment where premises are true, but conclusion is false.

Slide 7: Example of Valid Argument

- If John is an adult, then he is eligible to vote (premise).
- John is an adult (premise).
- Therefore, John is eligible to vote (conclusion).

Slide 8: Example of Valid Argument with False Conclusion

- If I catch the 19:32 train, I'll arrive in Glasgow at 19:53 (premise).
- I catch the 19:32 train (premise).
- Therefore, I arrive in Glasgow at 19:53 (conclusion) – Factually false but valid argument.

Slide 9: Example of Invalid Argument

- If I win the lottery, then I am lucky (premise).
- I do not win the lottery (premise).
- Therefore, I am unlucky (conclusion) – Invalid argument with factually true premises and conclusion.

Slide 10: Demonstrating Validity Using Truth Tables

- View argument as implication ($p \Rightarrow q$).
- If premises entail conclusion, then argument is valid.

Slide 12: Demonstrating Validity Using Truth Table (Example)

- Argument: If John is an adult, then he is eligible to vote; John is an adult; Therefore, John is eligible to vote.
- Atomic Propositions: p (John is an adult), q (John is eligible to vote).

p	q	$p \Rightarrow q$	$p \land q$
T	T	T	T
F	T	F	F

- Argument is valid because conclusion (q) is always true when premises are true.

Slide 13: Viewing Argument as Implication

- If premises logically imply conclusion, argument is valid.
- Example: $((p \Rightarrow q) \land p) \Rightarrow q$

Slide 15: Demonstrating Invalidity Using Truth Tables

- Argument is invalid if there's at least one assignment where premises are true, but conclusion is false.

Slide 16: Demonstrating Invalidity Using Truth Table (Example)

- Argument: p ⇔ q; p ⇒ r; Therefore, p – Invalid argument.

p	q	r	p ⇔ q	p ⇒ r
T	T	T	T	T
F	T	F	F	F

- Argument is invalid because there's a row where premises are true, but conclusion (p) is false.

Slide 17: Exercise

- Demonstrate the invalidity of the argument: p ∨ q; ¬p; Therefore, ¬q.

Slide 18: Solution to Exercise

- Atomic Propositions: p, q.

p	q	p ∨ q	¬p
F	T	T	T

- Argument is invalid because there's a row where premises are true, but conclusion (¬q) is false.

Slide 19: A Problem with Truth Tables

- Using truth tables to establish validity becomes tedious as the number of variables increases.

Slide 20: Deductive Proofs

- Approach to establishing validity using a series of simpler arguments known to be valid.
- Uses laws of logic (logical equivalences) and inference rules.

Slide 21: Inference Rules

- Primitive valid argument forms eliminating or introducing logical connectives.
- Categories: Intro (introduces connective), Elim (eliminates connective).

Slide 22: The Layout of an Inference Rule

- Premises (above the line): List of formulas already in proof.
- Conclusion (below the line): What may be deduced by applying the inference rule.

Slide 23: Conjunction (∧Intro)

- Introduces the connective ∧.
- Example: p, q; Therefore, p ∧ q.

Slide 24: Simplification (∧Elim)

- Eliminates the connective ∧.
- Example: p ∧ q; Therefore, p.

Slide 25: Addition (∨Intro)

- Introduces the connective ∨.
- Example: p; Therefore, p ∨ q.

Slide 26: Exercise on Disjunctive Syllogism

- Demonstrate the validity of the inference rule using a truth table.

Slide 27: Solution to Exercise

- Atomic Propositions: p, q.

p	q	¬p
F	T	T

- Argument is valid because conclusion (q) is always true when premises are true.

Slide 28: Modus Ponens (⇒Elim)

- Eliminates the connective ⇒.
- Example: p ⇒ q; p; Therefore, q.

Slide 29: Modus Tollens (⇒Elim)

- Eliminates the connective ⇒.
- Example: p ⇒ q; ¬q; Therefore, ¬p.

Slide 30: Other Inference Rules

- Double Negation (¬Elim): ¬¬p; Therefore, p.

- Laws of Equivalence (⇔Elim): $p \Leftrightarrow q$; Therefore, $p \Rightarrow q$ and $q \Rightarrow p$.

Slide 31: Transitive Inference Rules

- Transitivity of Equivalence: If $p \equiv q$ and $q \equiv r$, then $p \equiv r$.
- Hypothetical Syllogism: If $p \Rightarrow q$ and $q \Rightarrow r$, then $p \Rightarrow r$.

Slide 32: Summary

- Valid arguments: Conclusion always true when premises are true.
- Invalid arguments: At least one assignment where premises are true, but conclusion is false.
- Truth tables demonstrate invalidity.
- Inference rules deduce true propositions from other true propositions.

Slide 33: Reading and References

- Russell, Norvig (2022). Artificial Intelligence. 4th Edition.
- Nissanke (1999). Introductory Logic and Sets for Computer Scientists.
- Gray (1984). Logic, Algebra and Databases.