**Detailed Notes on Lectures 9 & 10: Validity and Inference Rules**

**Slide 1: Learning Objectives** - Define the notion of validity in an argument. - Establish validity using truth tables. - Demonstrate invalidity using truth tables. - Understand inference rules.

**Slide 2: Contents** - Objectives - Transformational proofs are not sufficient. - Comparison of deduction with induction. - Validity. - Demonstrating validity/invalidity using truth tables. - Problem with truth tables. - Inference rules. - Summary, reading, and references.

**Slide 3: Transformational Proofs do not Suffice** - Understanding transformations of formulas is useful but insufficient. - Logic uses rules of inference to deduce true propositions from other true propositions. - Invalid premises cannot lead to valid conclusions, preventing proofs of contradictions or useless systems.

**Slide 4: Premises and Conclusions** - An argument consists of premises (basis for accepting) and a conclusion. - Example: - Premises: Every adult is eligible to vote; John is an adult. - Conclusion: Therefore, John is eligible to vote.

**Slide 5: Deduction vs. Induction** - Deductive arguments: Conclusion is wholly justified by premises. - Inductive arguments: More general new knowledge inferred from facts or observations.

**Slide 6: Valid vs. Invalid Arguments** - Valid arguments: Conclusion always true when premises are true. - Invalid arguments: At least one assignment where premises are true, but conclusion is false.

**Slide 7: Example of Valid Argument** - If John is an adult, then he is eligible to vote (premise). - John is an adult (premise). - Therefore, John is eligible to vote (conclusion).

**Slide 8: Example of Valid Argument with False Conclusion** - If I catch the 19:32 train, I'll arrive in Glasgow at 19:53 (premise). - I catch the 19:32 train (premise). - Therefore, I arrive in Glasgow at 19:53 (conclusion) – Factually false but valid argument.

**Slide 9: Example of Invalid Argument** - If I win the lottery, then I am lucky (premise). - I do not win the lottery (premise). - Therefore, I am unlucky (conclusion) – Invalid argument with factually true premises and conclusion.

**Slide 10: Demonstrating Validity Using Truth Tables** - View argument as implication (p  q). - If premises entail conclusion, then argument is valid.

**Slide 12: Demonstrating Validity Using Truth Table (Example)** - Argument: If John is an adult, then he is eligible to vote; John is an adult; Therefore, John is eligible to vote. - Atomic Propositions: p (John is an adult), q (John is eligible to vote).

| p | q | p | q | p | q |
|---|---|---|---|---|---|
| T | T | T | | T | |
| F | T | F | | F | |

- Argument is valid because conclusion (q) is always true when premises are true.

**Slide 13: Viewing Argument as Implication** - If premises logically imply conclusion, argument is valid. - Example: ((p  q)  p)  q

**Slide 15: Demonstrating Invalidity Using Truth Tables** - Argument is invalid if there's at least one assignment where premises are true, but conclusion is false.

**Slide 16: Demonstrating Invalidity Using Truth Table (Example)** - Argument: p  q; p  r; Therefore, p – Invalid argument.

| p | q | r | p | q | p | r |
|---|---|---|---|---|---|---|
| T | T | T | T | | T | |
| F | T | F | F | | F | |

- Argument is invalid because there's a row where premises are true, but conclusion (p) is false.

**Slide 17: Exercise** - Demonstrate the invalidity of the argument: p  q; ¬p; Therefore, ¬q.

**Slide 18: Solution to Exercise** - Atomic Propositions: p, q.

| p | q | p | q | ¬p |
|---|---|---|---|---|
| F | T | T |  | T |

- Argument is invalid because there's a row where premises are true, but conclusion (¬q) is false.

**Slide 19: A Problem with Truth Tables** - Using truth tables to establish validity becomes tedious as the number of variables increases.

**Slide 20: Deductive Proofs** - Approach to establishing validity using a series of simpler arguments known to be valid. - Uses laws of logic (logical equivalences) and inference rules.

**Slide 21: Inference Rules** - Primitive valid argument forms eliminating or introducing logical connectives. - Categories: Intro (introduces connective), Elim (eliminates connective).

**Slide 22: The Layout of an Inference Rule** - Premises (above the line): List of formulas already in proof. - Conclusion (below the line): What may be deduced by applying the inference rule.

**Slide 23: Conjunction ( Intro)** - Introduces the connective  . - Example: p, q; Therefore, p  q.

**Slide 24: Simplification ( Elim)** - Eliminates the connective  . - Example: p  q; Therefore, p.

**Slide 25: Addition ( Intro)** - Introduces the connective  . - Example: p; Therefore, p  q.

**Slide 26: Exercise on Disjunctive Syllogism** - Demonstrate the validity of the inference rule using a truth table.

**Slide 27: Solution to Exercise** - Atomic Propositions: p, q.

| p | q | ¬p |
|---|---|---|
| F | T | T |

- Argument is valid because conclusion (q) is always true when premises are true.

**Slide 28: Modus Ponens ( Elim)** - Eliminates the connective  . - Example: p  q; p; Therefore, q.

**Slide 29: Modus Tollens ( Elim)** - Eliminates the connective  . - Example: p  q; ¬q; Therefore, ¬p.

**Slide 30: Other Inference Rules** - Double Negation (¬Elim): ¬¬p; Therefore, p. - Laws of Equivalence ( Elim): p  q; Therefore, p  q and q  p.

**Slide 31: Transitive Inference Rules** - Transitivity of Equivalence: If p  q and q  r, then p  r. - Hypothetical Syllogism: If p  q and q  r, then p  r.

**Slide 32: Summary** - Valid arguments: Conclusion always true when premises are true. - Invalid arguments: At least one assignment where premises are true, but conclusion is false. - Truth tables demonstrate invalidity. - Inference rules deduce true propositions from other true propositions.

**Slide 33: Reading and References** - Russell, Norvig (2022). Artificial Intelligence. 4th Edition. - Nissanke (1999). Introductory Logic and Sets for Computer Scientists. - Gray (1984). Logic, Algebra and Databases.